Nozzle Damping in Solid Rocket Instabilities

BEN T. ZINN*
Georgia Institute of Technology, Atlanta, Ga.

Present understanding regarding the damping provided by solid-propellant rocket motor nozzles, during axial instabilities, is reviewed. Expressions describing the various modes of wave energy losses to the nozzle and the nozzle decay coefficient are derived and their use in practice is discussed. Available theories for the prediction of the nozzle admittance and available methods for the experimental determination of nozzle admittances are evaluated. Experimental nozzle admittance data obtained by use of the impedance tube method, for two different solid-propellant rocket nozzles, is presented and discussed. An analysis of the experimental nozzle admittance data shows that 1) the admittances of short nozzles are independent of the frequency when the wavelength of the oscillation is much longer than the length of the nozzle convergent section, 2) the admittances of short nozzles are practically independent of the geometrical details of their convergent sections, and 3) the measured nozzle admittances are larger than those predicted by the short nozzle theory. The reported experimental data are used to derive expressions for the nozzle decay coefficients for cylindrical combustors experiencing axial instabilities. These expressions are compared with a corresponding expression derived in related experimental studies and good agreement is shown.

Nomenclature

A_N = nondimensional nozzle admittance defined in Eq. (11)
c = velocity of sound, fps
e = specific internal energy, Btu/lbm
E = energy flux per unit area, Btu/sec ft^2
h = specific internal enthalpy, Btu/lbm
H = specific stagnation enthalpy, Btu/lbm
i = imaginary unit, $(-1)^{1/2}$
I = mean wave energy flux per unit area, Btu/ft^2
Im = imaginary part of a complex quantity
J = ratio of nozzle throat area to chamber cross-sectional area
L = length, ft
M = mean flow Mach number
p = pressure, lbf/ft^2
r_c = chamber radius, ft
Re = real part of a complex quantity
s = specific entropy, Btu/(lbm °R)
S = cross-sectional area, ft^2; also nondimensional frequency, $\omega r_c/c_0$
t = time, sec
T = period of an oscillation, sec
u = axial component of velocity, fps
$\mathbf{v}$ = velocity vector, fps
$\hat{V}$ = quantity defined in Eq. (8)
Y = nozzle admittance, see Eq. (5), ft^3/sec lbf
z = axial distance, ft
α = growth or decay rate, sec^{-1}; also a measure of amplitude attenuation by the nozzle defined in Eq. (18)
β = quantity defined in Eq. (18)
γ = ratio of specific heats
Γ = nondimensional real part of nozzle specific admittance; defined in Eq. (20)
η = nondimensional imaginary part of nozzle specific admittance; defined in Eq. (20)
λ = wavelength of the oscillation, ft
Λ_N = nondimensional nozzle attenuation coefficient, $\alpha_N L_c/c_0$
ρ = density, lbm/ft^3
ω = frequency, rad/sec
$|\ |$ = absolute value of a quantity
$\langle\ \rangle$ = time average of a quantity, defined in Eq. (9)

Subscripts

0, 1, 2 = respectively, denote zeroth (i.e., steady state), first-, and second-order quantities
N = quantity related to the nozzle
c = quantity related to the combustion chamber

Superscript

$(\ )'$ = a perturbation quantity

Presented as Paper 72-1050 at the AIAA/SAE 8th Joint Propulsion Specialists Conference, New Orleans, La., November 29–December 1, 1972; submitted November 30, 1972, revision received June 27, 1973. The work described in this paper was partially supported by the Air Force Rocket Propulsion Laboratory under contract F04611-71-C-0054 and by the Naval Weapons Center, China Lake, Calif. The author gratefully acknowledges B. R. Daniel and B. A. Janardan and W. A. Bell for their cooperation and assistance throughout the course of the research that led to the preparation of this paper.

Index categories: Combustion Stability, Ignition, and Detonation; Solid and Hybrid Rocket Engines.

* Professor, School of Aerospace Engineering. Associate Fellow AIAA.

I. Introduction

THE susceptibility of solid-prepellant rocket motors to combustion instability depends upon the nature of the interactions between the flow disturbances and the various processes taking place inside the combustor and the nozzle. Some of these interactions, as the one with the combustion process, tend to increase the energy of the flow disturbances and thus exert a destabilizing influence upon the engine. Other interactions, such as those with the wave motion in the nozzle, with aluminum oxide particles in the combustor and so on, tend to dissipate the energy of the flow disturbances and thus exert a stabilizing influence upon the engine. Thus, performing a meaningful stability analysis of a solid-propellant rocket motor calls for an evaluation of the energy balance between the various disturbance (or wave) energy gains and disturbance energy losses that pertain to the engine under consideration. This paper is concerned with the evaluation of the effect of the exhaust nozzle upon the stability of solid-propellant rocket motors.

The nozzle damping depends upon the manner in which a disturbance generated inside the combustor interacts with the nozzle walls and the mean flow whose properties change along the nozzle. This complicated interaction process may perhaps be better understood if the nozzle flow is pictured as a collection of thin "pies," each characterized by a different cross-sectional area and different mean flow properties, that are attached to one another (see Fig. 1). As a wave moves from one of these pies to another, it is partially reflected and partially transmitted due to the change in mean flow properties and channel area. When the wave reaches the throat, where the flow is sonic, reflection can

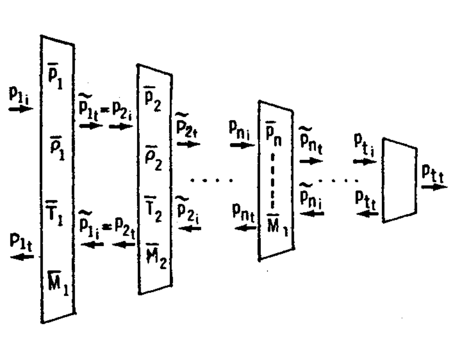


Fig. 1 Simplified schematic of wave propagation in nozzle.

no longer occur and the incident wave is carried out of the nozzle by the mean flow. The influence of these complicated wave reaches the throat, where the flow is sonic, reflection can the combustor is normally described by means of an appropriate boundary condition that describes the unsteady flow conditions at the nozzle entrance plane. The nozzle boundary condition is usually described by specifying the specific nozzle admittance Y that is defined as the complex ratio of the axial velocity perturbation to the pressure perturbation evaluated at the nozzle entrance. As will be shown shortly, the sign and magnitude of the real part of the nozzle admittance greatly influence the nature and magnitude of the acoustic energy flow across the nozzle entrance plane.

II. Analytical Considerations of Nozzle Losses

In this section expressions for the evaluation of the nozzle acoustic losses will be developed. Assuming that the flows in the combustor and nozzle are one-dimensional and isentropic, the instantaneous energy flux per unit area E crossing the nozzle entrance plane can be expressed in the following form:

$$\begin{aligned} E &= -(\rho\mathbf{u}\cdot\mathbf{n})(\tfrac{1}{2}u^2+e)-p\mathbf{n}\cdot\mathbf{u} \\ &= -(\rho\mathbf{u}\cdot\mathbf{n})(e+p/\rho+\tfrac{1}{2}u^2) \\ &= -(\rho\mathbf{u}\cdot\mathbf{n})(h+\tfrac{1}{2}u^2) = -(\rho\mathbf{u}\cdot\mathbf{n})H \end{aligned} \quad (1)$$

where $\mathbf{n}$ is a unit normal vector pointing from the nozzle into the combustor and ρ, $\mathbf{u}$, e, p, h, and H, respectively, denote the density, velocity vector, specific internal energy, pressure, specific enthalpy, and specific stagnation enthalpy. As can be seen from Eq. (1), the energy flux E consists of two parts; the energy convected by the flow and the work done by the pressure forces, at the nozzle entrance plane, upon the gases inside the nozzle; the latter is often referred to as "pumping" or displacement work.

Since wave energy is known[1] to be a second-order quantity, it is not sufficient to consider first-order quantities only in the evaluation of the wave energy flux into the nozzle. To evaluate the wave energy flux that is crossing the nozzle entrance plane, each of the quantities appearing in Eq. (1) must be expanded to second order, the resulting expression should be separated into its steady and unsteady parts and the unsteady part should be averaged over a long period of time. A typical second-order expansion takes on the following form:

$$\begin{aligned} \rho &= \rho_0+\rho' = \rho_0+\rho_1+\rho_2+\cdots \\ &= \rho(p_0, s_0, 0) + \left(\frac{\partial\rho(p_0, s_0, 0)}{\partial p}\right)_s \varepsilon \\ &\quad + \frac{1}{2}\left(\frac{\partial^2\rho(p_0, s_0, 0)}{\partial p^2}\right)_s \varepsilon^2 + \cdots \end{aligned} \quad (2)$$

The preceding series may be considered as Taylor series expansion about the unperturbed state (i.e., ρ_0). The actual evaluation of $\langle E-E_0\rangle$, the time average of the mean wave energy flux, requires the use of some physical arguments as well as the mass and momentum conservation laws. This is done in detail in Ref. 2, which arrives at the result that

$$\begin{aligned} \langle E-E_0\rangle = I &= \langle (p_1/\rho_0+\mathbf{u}_0\cdot\mathbf{u}_1)(\rho_0\mathbf{u}_1+p_1\mathbf{u}_0/c_0^2)\cdot\mathbf{n}\rangle = \\ &\langle \{p_1\mathbf{u}_1+p_1^2\mathbf{u}_0/\rho_0c_0^2+\rho_0(\mathbf{u}_0\cdot\mathbf{u}_1)\mathbf{u}_1+(\mathbf{u}_0\cdot\mathbf{u}_1)p_1\mathbf{u}_0/c_0^2\}\cdot\mathbf{n}\rangle \end{aligned} \quad (3)$$

The derivation of Eq. (3) required the assumption that the disturbance is either periodic or slowly varying with time. It is significant to point out that though I is a second-order quantity, it can be evaluated from knowledge of first-order quantities only. Thus, the evaluation of the mean wave energy flux does not require pursuing the usually difficult solution of any second-order equations. One should further notice that in the limit of zero mean flow (i.e., $\mathbf{u}_0 = 0$), Eq. (3) reduces to the well-known expression for sound intensity; that is, $I = \langle p_1\mathbf{u}_1\rangle$.

Letting the pressure and velocity perturbations at the nozzle entrance be

$$p_1 = Re\{|P_N|e^{i\omega t}\}, \quad u_1 = Re\{|U_N|e^{i(\omega t+\theta)}\} \quad (4)$$

where Re denotes the real part of a complex quantity and θ is the phase difference between the velocity and pressure perturbations, then the specific nozzle admittance Y can be expressed in the following form

$$Y = u_1/p_1 = [|U_N|/|P_N|]e^{i\theta} \quad (5)$$

Using Eqs. (4) and (5) the expression for I, Eq. (3), becomes

$$\begin{aligned} I = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} &\Big\{Re(|P_N|e^{i\omega t})Re(Y|P_N|e^{i\omega t})+ \\ &(u_0/\rho_0c_0^2)Re(|P_N|e^{i\omega t})^2+\rho_0u_0Re(Y|P_N|e^{i\omega t})^2+ \\ &(u_0^2/c_0^2)Re(|P_N|e^{i\omega t})Re(Y|P_N|e^{i\omega t}\Big\}dt = \\ &\tfrac{1}{2}|P_N|^2[(1+M_0^2)Re\{Y\}+(M_0/\rho_0c_0)+\rho_0c_0M_0|Y|^2] \end{aligned} \quad (6)$$

where $M_0 = u_0/c_0$ and $Re\{Y\} = |U_N|\cos\theta/|P_N|$.

Examination of Eq. (6) shows that when M_0 is small, which is the case in most solid-propellant rocket motors, the first term of the final expression for I can potentially dominate the remaining terms in this expression and thus control the wave energy transfer into or out of the nozzle. It should also be pointed out that this term, which is also referred to as the wave energy radiation term, is directly related to the displacement work term in Eq. (1). The sign and magnitude of this term depend upon the sign and magnitude of $Re\{Y\}$ which, respectively, depend upon $\cos\theta$ and $|U_N|\cos\theta/|P_N|$. Studies conducted to date[3] indicate that in the case of axial oscillations $Re\{Y\}$ is always positive, while in the case of transverse or three-dimensional oscillations[4,5] $Re\{Y\}$ may be either positive or negative depending upon the frequency of the wave and the geometry of the nozzle convergent section. The remaining terms in Eq. (6) are due to the convection of wave energy by the mean flow, and they are expected to be smaller than the abovementioned wave energy radiation term.

In practice (e.g., see Refs. 6 and 7), the effect of the nozzle upon the stability of solid-propellant rocket motors is usually determined by evaluating the nozzle decay coefficient α_N that provides a measure of the exponential attenuation of a small-amplitude disturbance, provided by a given nozzle design, at given chamber conditions, when the nozzle is the only means of transferring wave energy into or out of the combustor under consideration. A relationship for the evaluation of α_N can be obtained from the analysis of Ref. 8 where the behavior of a small-amplitude disturbance, in a combustor with mean flow and wave energy addition and/or removal at the combustor's boundaries only, is analyzed. According to Ref. 8, the growth or decay rate α of a small-amplitude disturbance can be expressed in the following general form:

$$2\alpha\hat{V} = -\left\langle \int_{\substack{\text{Combustor}\\ \text{Boundary}}} dS\mathbf{n}\cdot\left\{p_1\mathbf{v}_1+p_1^2\frac{\mathbf{v}_0}{\rho_0c_0^2}+ \rho_0(\mathbf{v}_0\cdot\mathbf{v}_1)\mathbf{v}_1+\frac{p_1}{c_0^2}(\mathbf{v}_0\cdot\mathbf{v}_1)\mathbf{v}_0\right\}\right\rangle \quad (7)$$

where

$$\hat{V} = \left\langle \int_{\substack{\text{Combustor}\\ \text{Volume}}} \left\{\frac{1}{2}\rho_0\mathbf{v}_1\cdot\mathbf{v}_1+\frac{1}{2}\frac{p_1^2}{\rho_0c_0^2}+\frac{\mathbf{v}_0\cdot\mathbf{v}_1}{c_0^2}p_1\right\}dV\right\rangle \quad (8)$$

and

$$\langle\ \rangle = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} (\ )\,dt \tag{9}$$

To obtain an expression for the nozzle decay coefficient α_N it is necessary to assume that removal or addition of wave energy occurs at the nozzle entrance only. Under these conditions the surface integration in Eq. (7) is replaced by integration over the nozzle entrance area only yielding, for the case of a one-dimensional oscillation, the following result

$$2\alpha_N \hat{V} = -S_N I \tag{10}$$

Where S_N is the nozzle entrance area and I is given by Eq. (6). Equation (10) clearly demonstrates the relationship between the wave energy flux across the nozzle entrance plane and the nozzle decay coefficient. When the nozzle removes wave energy from the combustor, I is positive, and since $\hat{V}$ is positive, then α_N must be negative and the chamber disturbance decays with time; the reverse occurs when the interaction of the disturbance with the nozzle results in wave energy transfer into the combustor.

In some investigations of combustion instability in solid-propellant rocket motors (e.g., see Ref. 6) the following definition of nozzle admittance is used

$$A_N = \frac{\gamma p_0}{u_0}\frac{u_1}{p_1} = \frac{\gamma p_0}{u_0} Y \tag{11}$$

Substituting Eq. (11) into Eq. (10) and using Eq. (6) yields the following nondimensional equation

$$\frac{4\rho_0 c_0 \alpha_N \hat{V}}{S_N |P_N|^2} = -M_0\{(1+M_0{}^2)Re\{A_N\}+1+M_0{}^2|A_N|^2\} \tag{12}$$

that may also be used to evaluate α_N. When $|A_N|$ is of order one and the mean flow Mach number at the nozzle entrance is small, Eq. (12) may be further simplified to yield the following approximate result

$$\frac{4\rho_0 c_0 \alpha_N \hat{V}}{S_N |P_N|^2} = -M_0(Re\{A_N\}+1) \tag{13}$$

Examination of Eqs. (6–13) shows that the evaluation of α_N requires an a priori knowledge of the precise wave structure inside the combustor, the mean flow properties throughout the combustor and the nozzle admittance. Furthermore, Eq. (7) indicates that precise determination of α_N requires that the combustor mean flow properties, the wave mode structure and the nozzle admittance be known under the same growth or decay conditions as those experienced by a disturbance in the actual combustor. Even if the dependence of the wave mode structure and the nozzle admittance upon the growth rate constant α is negligible, the complexity of both the mean flow conditions inside the combustor and the combustor geometry render the task of precisely determining the wave structure, by any means, a most difficult if not an impossible one. In view of these comments, computed values of the nozzle decay coefficient should be viewed as only an approximate measure of the damping provided by the nozzle. The problem of determining the combustor's mode structure requires detailed analysis or measurements of the acoustic properties of the chamber and it lies beyond the scope of this paper. In what follows, the current state of knowledge regarding the nozzle admittance will be discussed.

III. Theoretical Prediction of Nozzle Admittance

Theoretical treatments for computing the nozzle admittance are complex in nature, and to date, solutions have been obtained for only a limited number of cases. The most sophisticated treatment of the nozzle admittance problem was provided by Crocco[9] who analyzed the behavior of both three- and one-dimensional wave motions in nozzles whose mean flows are one-dimensional. The latter requirement restricts this analysis to slowly converging nozzles. This theory[9] indicates that the nozzle admittance depends upon the gradients of the mean flow properties in the convergent section of the nozzle, the frequency, the amplification factor, the mode of oscillation, and the magnitude of the mean flow Mach number at the nozzle entrance.

Examination of the predictions[4,9] of Crocco's nozzle admittance theory shows that when the ratio of the length of the nozzle convergent section to the wavelength of the oscillation is small, the predicted nozzle admittance is practically independent of the frequency of the oscillation. Under these conditions the ratio of the wave travel time in the nozzle to the wave travel time in the combustor is very small, and the incident and reflected pressure waves are practically in phase at the nozzle entrance. Nozzles satisfying these conditions are in general referred to as being "quasi-steady," and their responses may be predicted by analyzing the steady-state conservation equations. It should, however, be pointed out that quasi-steadiness does not mean that the nozzle flow properties do not change with time. On the contrary, quasi-steadiness implies that these properties change very fast and they respond almost instantaneously to any changes that are taking place at the nozzle entrance.

Since the lengths of the convergent sections of many practical solid-propellant rocket nozzles are considerably shorter than characteristic combustor dimensions, these nozzles may be considered to be quasi-steady. The response to quasi-steady nozzles was analyzed in detail in Ref. 10 and later reviewed in Ref. 2. Using the steady-state conservation equations for a perfect gas and assuming that the flow between the nozzle entrance plane† and the throat is both isoenergetic and isentropic, Ref. 10 derives the following expression for the nozzle admittance Y

$$Y = \frac{u_1}{p_1} = \left(\frac{\gamma-1}{2\gamma}\right)M_0\frac{c_0}{p_0} = \left(\frac{\gamma-1}{2\gamma}\right)\frac{u_0}{p_0} \tag{14}$$

The applicability of Eq. (14) will be checked in the following section where its predictions will be compared with available experimental data.

IV. Experimental Determination of Nozzle Damping

To date several experimental techniques have been used to determine the damping capabilities of rocket nozzles. These are often referred to as the direct, wave-attenuation, frequency response, and standing wave methods. A brief description of each of these methods follows.

1) In the direct technique[11] a hot-wire anemometer probe and a pressure transducer are installed in the entrance plane of the nozzle. The nozzle admittance is determined from direct measurements of the amplitude and phases of the axial flow velocity and pressure oscillations at the nozzle entrance. However, experimental difficulties and inaccuracies that are associated with the measurements of the velocity perturbation reduce the quality of the measured nozzle admittances.

2) In the wave-attentuation or decay technique[12] a pressure pulse is superimposed on a steady flow in a simulated chamber by bursting a diaphragm at the upstream end of the chamber. The decay rate of the resulting pressure pulse is measured, and this data is used to determine the damping capability of the nozzle. This technique is limited to investigations of nozzles having relatively small values of the ratio of the nozzle throat area to the chamber cross-sectional area because large values of this ratio result in extremely fast damping and high background flow noise. This method assumes that the observed decay is solely due to nozzle damping; a rather questionable assumption. Since the decaying pulse oscillates at the resonant frequency of the chamber, the measured admittance is only applicable to this particular frequency. To evaluate the admittance at other frequencies the chamber length must be modified; a requirement that considerably detracts from the attractiveness of this method

† In the quasi-steady analysis the nozzle entrance plane is taken to be at a location a short distance upstream of the actual nozzle entrance where the mean flow separates from the chamber wall.

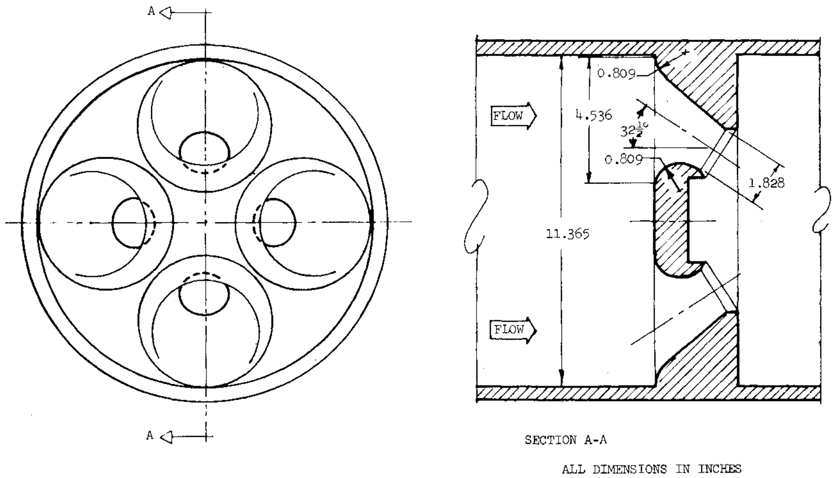


Fig. 2 A typical solid-propellant rocket four-ported nozzle tested in Ref. 3.

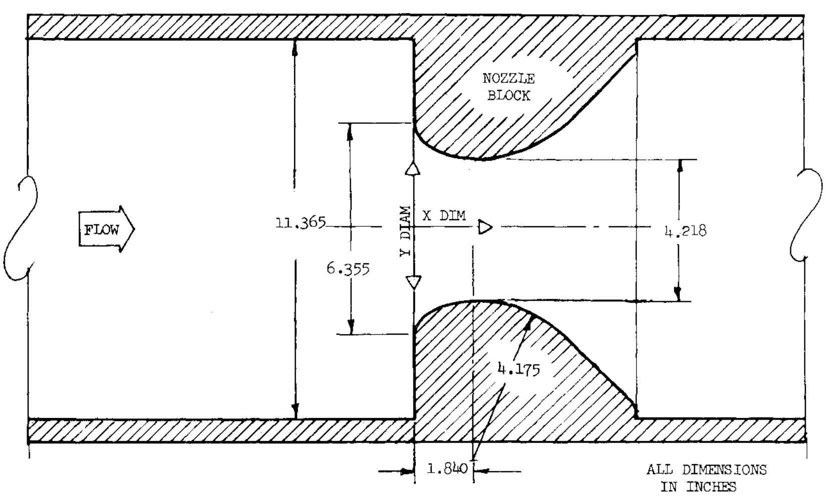
in situations where nozzle response over a wide frequency range is needed.

3) In the frequency response or steady-state resonance technique[12] pressure waves of different frequencies are generated within the chamber and the amplitudes of the resulting pressure oscillations are measured. These data can then be used to determine the nozzle admittance at the resonant frequency by a procedure described in Ref. 13. This method suffers from most of the shortcomings that limit the applicability of wave-attenuation methods.

4) The standing wave or modified impedance tube technique is an extension of the classical, no flow, impedance tube method[1,14] long employed to measure the acoustical damping capabilities of various materials. In the classical impedance tube technique, a sound source capable of generating a wave of desired waveform and frequency is placed at one end of a tube. The other end of the tube is terminated with a sample of the material whose damping capabilities, or admittance, are to be measured. A standing wave pattern of a given frequency is excited in the tube, and a microphone probe is traversed along the length of the tube to measure the axial variation of the standing wave amplitude. The damping capability of the material can then be determined by measuring the distance of the first pressure amplitude minimum or maximum from the tested sample and the ratio of the maximum pressure amplitude to the minimum pressure amplitude. The frequency dependence of the damping of the tested sample may be determined by repeating the experiment at different frequencies. In this experiment, changing the frequency merely involves changing the frequency of the generated wave; a process that is considerably simpler than the modifications required in the wave-attenuation and resonance experiments.

The accuracy and applicability of some of these experimental techniques were analyzed by Culick and Dehority in Ref. 13. One of the main conclusions of this study was that the impedance tube method appears to be the most suitable method for the measurement of nozzle admittances. A similar conclusion was reached independently in the work described in Ref. 4. References 3, 4, and 13 derive the equations and discuss the experimental setup that are needed for the above-mentioned impedance tube experiments. To date, the impedance tube method has been applied in the determination of the damping provided by liquid-propellant rocket nozzles subjected to three-dimensional pressure oscillations[4,5] and the damping provided by various solid-propellant rocket nozzles subjected to axial oscillations.[3] In what follows some of the experimental admittance data obtained in the investigations described in Ref. 3 will be discussed and compared with available theoretical predictions.

The nozzles whose measured admittance will be discussed in this section are shown in Figs. 2 and 3 where these nozzles are shown as they appear in the modified impedance tube



STA	0	1	2	3	4	5	6	THROAT
X DIM	0	0.077	0.153	0.307	0.537	0.843	1.166	1.840
Y DIAM	6.355	5.680	5.410	5.061	4.736	4.472	4.294	4.218

Fig. 3 A typical solid-propellant rocket single-ported nozzle tested in Ref. 3.

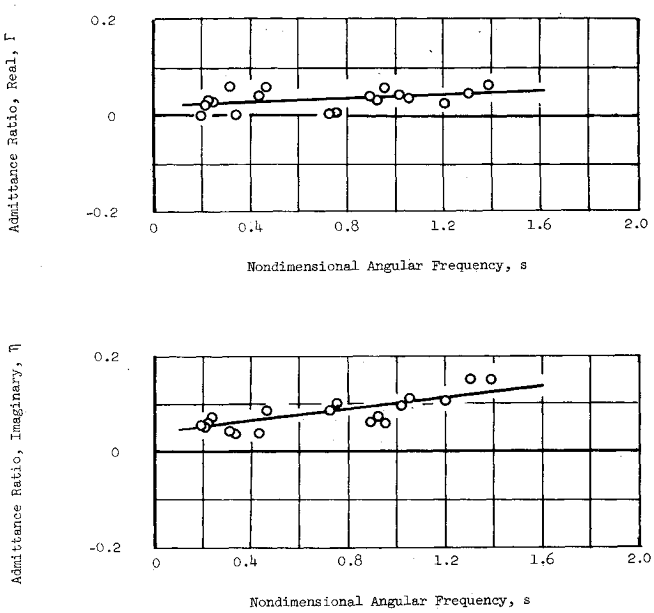


Fig. 4 Frequency dependence of the nondimensional real and imaginary parts of the admittance of the nozzle shown in Fig. 2.

setup. These small scale nozzles were designed to simulate the geometries of actual solid-propellant nozzles as well as the mean flow conditions at the entrance planes of the actual nozzles. Examination of the nozzles shown in Figs. 2 and 3 shows that these nozzles have short convergent sections; hence, since one can show that these lengths are considerably shorter than the wavelengths of the oscillations observed during axial instabilities, one would expect that these nozzles would respond in a quasi-steady manner during axial instabilities.

The admittances of the nozzles shown in Figs. 2 and 3 were determined by use of the modified impedance tube technique. The results of these tests are shown in Figs. 4 and 5 where the nondimensional real and imaginary parts of the nozzle admittances Γ and η, defined as

$$\Gamma = Re\left\{\frac{\rho_0 c_0}{g}\frac{u_1}{p_1}\right\}, \qquad \eta = Im\left\{\frac{\rho_0 c_0}{g}\frac{u_1}{p_1}\right\} \tag{15}$$

are plotted vs the nondimensional frequency $S = \omega r_c/c_0$. Examination of these figures indicates that in the range of frequencies under investigation the measured admittances are indeed independent of the frequency; an observation which is in agreement with the above expectation and the theoretical predictions of Ref. 10. A comparison of the magnitudes of the measured nozzle admittances with the corresponding theoretical predictions, provided by Eq. (14), is shown in Table 1. The data in Table 1 shows that the short-nozzle theory[10] underestimates the magnitudes of the real and imaginary parts of the nozzle admittance. The discrepancy in the measured and predicted values of η reflects the limitation of the quasi-steady assumption used in the development of the short-nozzle theory; the reasons for the disagreement between the measured and predicted values of Γ are not clear at this time.‡

Table 1 Comparison of experimental and theoretical nozzle admittance values

Type of nozzle	Entrance Mach number	Real Part of admittance, Γ		Imaginary part of admittance, η	
		Measured	Theoretical	Measured	Theoretical
Four-nozzle combination (Figs. 2 & 4)	0.06	0.044	0.012	0.09	0
Single nozzle (Figs. 3 & 5)	0.08	0.047	0.016	0.03	0

Although the geometrical features of the nozzles shown in Figs. 2 and 3 are considerably different from one another, their measured admittances are close to one another. This observation together with the data presented in Ref. 3 suggest that when the condition $L_N/\lambda \ll 1$ is satisfied, the nozzle response is independent of the geometrical details of the nozzle convergent section. The smallness of the measured values of Γ, for both nozzles, indicates that these nozzle designs remove little acoustic energy from the combustor; consequently, these nozzles can be expected to provide little damping for axial instabilities.

Further experimental investigations that are presently in progress, as a continuation of the efforts reported in Ref. 3, are concerned with the experimental determination of 1) the damping of submerged nozzles and the dependence of this damping upon the depth of the cavity surrounding the nozzle and the magnitude of the secondary flow rate issuing from this cavity, 2) the scaling laws that should be employed in scaling nozzle admittance data obtained in cold flow, small scale experiments, 3) the dependence of nozzle damping upon the number of nozzles present in the nozzle cluster, and 4) the dependence of nozzle damping upon the shape of the nozzle convergent section. The results of these studies will be reported in a future publication.

V. Application and Discussion of the Data

As an illustration of how the data presented in this paper may be used to estimate the nozzle damping, the nozzle decay coefficients for the modified impedance tube, that would result from the use of either of the nozzles shown in Figs. 2 and 3, will be evaluated. When both the constant mean flow and the oscillation in the impedance tube are one-dimensional, the solutions describing the behavior of the pressure and velocity perturbations may be expressed in the following form[3,4]:

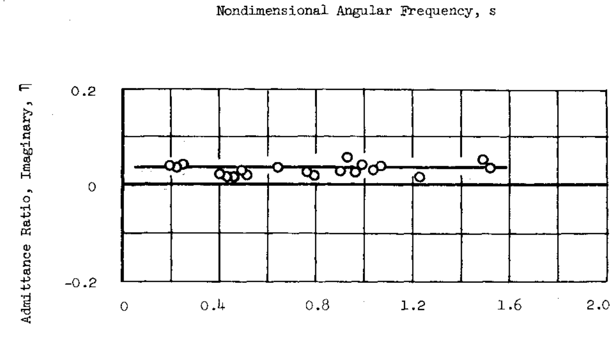


Fig. 5 Frequency dependence of the nondimensional real and imaginary parts of the admittance of the nozzle shown in Fig. 3.

‡ Recent analysis of this problem suggests that the observed discrepancy may be, in part, due to the neglect of entropy waves in the short nozzle theory; this problem is presently under study.

$$p_1(z,t) = \gamma p_0 A\, e^{i(\omega t + az)} \cosh(\Phi - iRz)$$
$$u_1(z,t) = c_0 A\, e^{i(\omega t + az)} \sinh(\Phi - iRz) \tag{16}$$

where

$$a = \omega M / c_0(1-M^2), \qquad R = a/M$$
$$\Phi = \pi\alpha - i\pi\left[\beta + \frac{2(L_c - z)}{\lambda} + \frac{1}{2}\right] \tag{17}$$

and

$$\lambda = 2\pi c_0(1-M^2)/\omega$$

It can also be shown[4] that the parameters α and β, appearing in Φ, respectively describe the change in amplitude and phase between the incident and reflected pressure waves at the nozzle entrance; i.e.,

$$\left.\frac{\text{reflected pressure wave amplitude}}{\text{incident pressure wave amplitude}}\right|_{\substack{\text{Nozzle}\\ \text{entrance}}} = e^{-2\pi\alpha} \tag{18}$$

$$\left\{\begin{matrix}\text{Phase change between incident and reflected}\\ \text{pressure waves at nozzle entrance}\end{matrix}\right\} = \pi(2\beta + 1)$$

where the parameter β must satisfy the condition $|\beta| \leq 0.5$.

To proceed with the evaluation of α_N, the real parts of u_1 and p_1, given in Eq. (16), must be substituted into Eqs. (7) and (8) and the indicated space integration and time averaging must be performed. These somewhat lengthy manipulations[3] yield the following expression for the nondimensional nozzle decay coefficient Λ_N:

$$\Lambda_N = \frac{\alpha_N L_c}{c_0} = -\frac{M + [(1+M^2)/2]\tanh 2\pi\alpha}{1 + M\tanh 2\pi\alpha} \tag{19}$$

Inspection of Eq. (19) shows that in the case under consideration, the nondimensional nozzle decay coefficient Λ_N is a function of only the mean flow Mach number and the parameter α that according to Eq. (18) describes the amplitude attenuation provided by the nozzle. As shown in Refs. 3 and 4, the parameters α and β are directly related to Γ and η through the relationships

$$\Gamma = \frac{\tanh(\pi\alpha)\sec^2(\pi\beta)}{\tanh^2(\pi\alpha) + \tan^2(\pi\beta)}$$

and (20)

$$\eta = \frac{\operatorname{sech}^2(\pi\alpha)\tan(\pi\beta)}{\tanh^2(\pi\alpha) + \tan^2(\pi\beta)}$$

From the preceding expressions for Γ and η it can be shown that when these quantities are small, as is the case with the data presented in Table 1, then $\Gamma \approx \pi\alpha$ and $\beta \approx 0.5$. Using these results together with the assumption that $M \ll 1$, reduce Eq. (19) to the following approximate form:

$$\Lambda_N = -(M + \Gamma) \tag{21}$$

To relate the above result to nozzle geometry and to allow for comparison with other experimental data[12] the following approximate relationship is introduced[2]:

$$M \approx [2/(\gamma+1)]^{(\gamma+1)/2(\gamma-1)} J \tag{22}$$

In the case of air $\gamma = 1.4$ and Eqs. (21) and (22) may be combined to give

$$\Lambda_N = -[0.58 + (\Gamma/J)]J \tag{23}$$

Using the Mach number data given in Table 1 to evaluate the values of J and substituting these values together with the measured values of Γ into Eq. (23) yield the following expressions for the nondimensional nozzle decay coefficients that result from the use of the nozzles shown in Figs. 2 and 3.

$$\text{Four-ported nozzle } (M = 0.06): \Lambda_N = -1.005\, J$$
$$\text{Single-ported nozzle } (M = 0.08): \Lambda_N = -0.922\, J \tag{24}$$

These results are in good agreement with the results of Ref. 12 which uses nozzle damping data obtained by use of the steady-state resonant method and the pressure decay method to arrive at the following empirical result:

$$\Lambda_N = -J \tag{25}$$

By comparison, the use of the results of the short-nozzle theory[2] yields the approximate result

$$\Lambda_N \approx -0.69\, J \tag{26}$$

A comparison of the results presented in Eqs. (24–26) shows good agreement between the experimental sets of data and a difference between the experimental and theoretical results. The latter comparison indicates that the available short-nozzle theory underestimates the damping provided by short nozzles. It is interesting to note that the good agreement between the experimental data was obtained in spite of the fact that the tested nozzles had entirely different geometries and in spite of the fact that they were tested by use of entirely different experimental techniques. The first part of the above observation further supports the conclusion that the damping provided by short nozzles is independent of the geometrical details of the nozzle designs and primarily depends upon the nozzle area ratio J. If this conclusion is indeed correct, then the future evaluation and prediction of the damping provided by short nozzles will be considerably simplified.

References

[1] Morse, P. M. and Ingard, K. U., *Theoretical Acoustics*, McGraw Hill, New York, 1968.

[2] Zinn, B. T., "Longitudinal Mode Acoustic Losses in Short Nozzles," *Journal of Sound and Vibrations*, Vol. 22, No. 1, 1972, pp. 93–105.

[3] Zinn, B. T., Daniel, B. R., Janardan, B. A., and Smith, A. J., Jr., "Damping of Axial Instabilities by the Minuteman II, Stage III, and Minuteman III, Stage III, Exhaust Nozzles," AFRPL-TR-72-71, Aug. 1972, Air Force Rocket Propulsion Lab., Edwards, Calif.

[4] Bell, W. A., "Experimental Determination of Three-Dimensional Liquid Rocket Nozzle Admittances," Ph.D. thesis, July 1972, Georgia Inst. of Technology, Atlanta, Georgia.

[5] Zinn, B. T., Bell, W. A., Daniel, B. R., and Smith, A. J., Jr., "Experimental Determination of Three Dimensional Liquid Rocket Nozzle Admittances," *AIAA Journal*, Vol. 11, No. 3, March 1973, pp. 267–272.

[6] "The Investigation of Pressure Oscillations During Firing of the Minuteman II Stage III Motor," Hercules Rept. MTO-1124-34, Jan. 1971, Hercules Corp., Bacchus, Utah.

[7] "Minuteman III Third Stage Pressure Oscillation Study," Aerojet Rept. 1387-01F, Aug. 1971.

[8] Cantrell, R. H. and Hart, R. W., "Interaction Between Sound and Flow in Acoustic Cavities; Mass, Momentum and Energy Considerations," *Journal of the Acoustical Society of America*, Vol. 36, No. 4, April 1964, pp. 697–706.

[9] Crocco, L. and Sirignano, W. A., "Behavior of Supercritical Nozzles under Three-Dimensional Oscillatory Conditions," AGARDograph 117, 1967, Princeton Univ., Princeton, N.J.

[10] Crocco, L. and Sirignano, W. A., "Effect of Transverse Velocity Components on the Nonlinear Behavior of Short Nozzles," *AIAA Journal*, Vol. 4, No. 8, Aug. 1966, pp. 1428–1430.

[11] Crocco, L., Monti, R., and Grey, J., "Verification of Nozzle Admittance Theory by Direct Measurement of the Admittance Parameter," *ARS Journal*, Vol. 31, June 1961, pp. 771–775.

[12] Buffum, F. G., Dehority, G. L., Slates, R. O., and Price, E. W., "Acoustic Attenuation Experiments on Subscale, Cold-Flow Rocket Motors," *AIAA Journal*, Vol. 5, No. 2, Feb. 1967, pp. 272–280.

[13] Culick, F. E. C. and Dehority, G. L., "An Analysis of Axial Acoustic Waves in a Cold Flow Rocket," Rept. 4544, Naval Weapons Center, China Lake, Calif., May 1968.

[14] Beranek, L. L., *Acoustic Measurements*, Wiley, New York, 1949.